Regulæ
variabiliter et independenter differentiandi generales,

quas

Consent. Ampliss. Ord. Philosoph.,

Præside

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altissimum ($d^r x$) variandum est, et quidem singulari, quem docuimus, modo (per $\overset{1}{\underset{c}{d}}$ indicato) differentiando. Hæ vero formulæ regulas jam præscriptas breviter exprimunt. Patet vero, calculatorem excercitatum differentiationem, multiplicationem & divisionem, si has regulas aliquando bene perspexerit, junctim seu uno actu perficere posse, ut ita differentiale quæsitum currente calamo perscribat. Præterea observamus, in terminis extremis ($dx^n \partial^n f$ et $d^n x \partial f$) coefficientes unitatem æquare, ideoque in his efformandis multiplicationem et divisionem, a regula præscriptas, omnino omitti posse; reliquosque semper fore numeros integros.

7) Ex regulis jam traditis $d^n f$ ab $d^{n-1} f$, ideoque ut dicunt *recurrenter*, derivatur, at utrumque absolute, (seu a se invicem *independenter*, ut dicunt) describendi ars quæri potest.

Jam vero primum ipsius $d^n f$ terminum $(dx)^n \partial^n f$ obtinuimus, indeque ipsius $d^{n+1} f$ duo priores $\widehat{dx}^{n+1}.\partial^{n+1} f + \frac{\widehat{n.n+1}}{2} \widehat{dx}^{n-1} d^2 x \partial^n f + \ldots\ldots$ ex regula præscripta facile elicies: tum igitur inde, tum ex ipsis exemplis jam allatis, colligitur, hanc ipsius $d^n f$ fore formam:

$$d^n f = X_0 \partial^n f + X_1 \partial^{n-1} f + X_2 \partial^{n-2} f + X_3 \partial^{n-3} f + \ldots$$

ubi X_0, X_1, X_2, et reliqui coefficientes per dx, d^2x, d^3x etc. exprimuntur; at quomodo? Ex modo dictis quidem colligitur $X_0 = \widehat{dx}^n$, atque

$$X_1 = \frac{\widehat{n.n-1}}{2} \widehat{dx}^{n-2} d^2 x, \text{ seu } X_1 = \overset{1}{\underset{c}{d}}(n\widehat{dx}^{n-1}), \text{ vel etiam } X_1 = \underset{1}{\underset{c}{d}}(\partial \widehat{dx}^n) = \underset{1}{\underset{c}{d}}\, \partial \widehat{X}_0,$$

indicante $\underset{1}{\partial}$ derivationem secundum dx; X_1 igitur ex X_0 (coefficiente anteriore) derivatur, et quidem hunc prævie secundum dx vulgari modo derivando deindeque singulariter ut antea differentiando; quæ quidem regula generalis est: *ut enim X_{r+1} ab X_r obtineatur, aliud haud opus est, quam*

$(\underset{1}{\delta} X_r =)$ *ipsum* X_r *jam explicitum secundum* dx *derivare* (i. e. $s\widehat{dx}^{s-1}$ loco $\widehat{dx}^{s}$ ubique scribere), *quodque ita exoritur, secundum regulam jam traditam* ($\overset{1}{\underset{c}{d}}$) *singulariter differentiare.* Id quod ita significari potest: $X_{r+1}=\overset{1}{\underset{c}{d}}(\underset{1}{\delta}X_r)$. Ita Ex. gr. ex $X_0=\widehat{dx}^n$ obtinetur primum $\underset{1}{\delta}X_0=n\widehat{dx}^{n-1}$, & deinde $\overset{1}{\underset{c}{d}}(dx)^{n-1}=\frac{n-1}{2}\widehat{dx}^{n-2}d^2x$, atque igitur $X_1=\overset{1}{\underset{c}{d}}(n.\widehat{dx}^{n-1}) = \frac{\widehat{n.n-1}}{2}\widehat{dx}^{n-2}d^2x$, ut jam observavimus. Tum ex $X_1=\frac{\widehat{n.n-1}}{2}\widehat{dx}^{n-2}d^2x$ oritur primum $\underset{1}{\delta}X_1=\frac{\widehat{n.n-1}}{2}.\widehat{n-2}\,\widehat{dx}^{n-3}d^2x$, deindeque

$$X_2=\overset{1}{\underset{c}{d}}(\underset{1}{\delta}X_1)=\frac{\widehat{n.n-1}}{2}.\widehat{n-2}.\left(\frac{\widehat{n-3}}{2.2}.\widehat{dx}^{n-4}\,\widehat{d^2x}^2+\widehat{dx}^{n-3}.\frac{d^3x}{3}\right).$$

Ulteriusque $\underset{1}{\delta}X_2=\frac{\widehat{n.n-1}}{2}.\widehat{n-2}.\left(\frac{\widehat{n-3}}{4}.\widehat{n-4}\,\widehat{dx}^{n-5}(d^2x)^2+\frac{\widehat{n-3}}{3}.\widehat{dx}^{n-4}.d^3x\right)$, indeque $X_3=\overset{1}{\underset{c}{d}}(\underset{1}{\delta}X_2)=\frac{\widehat{n.n-1}}{2}.\widehat{n-2}.\widehat{n-3}\frac{\widehat{n-4}}{4}.\left(\frac{\widehat{n-5}\,\widehat{dx}^{n-6}\,\widehat{d^2x}^3}{2.3}+\widehat{dx}^{n-5}.2d^2x.\frac{d^3x}{3}\right)+\frac{n}{2}.\widehat{n-1}.\widehat{n-2}.\frac{\widehat{n-3}}{3}.\widehat{dx}^{n-4}.\frac{d^4x}{4}.$

Eundemque in modum successive calculabis X_4, X_5, X_6 etc.

Positis igitur $X_0=(dx)^n$, (et n_r (ut $n_2=\frac{\widehat{n.n-1}}{2}$) = coefficiente binomii erit $X_1=n_2(dx)^{n-2}d^2x$ seu breviter

$$X_1=\overset{1}{\underset{c}{d}}(\underset{1}{\delta}X_0)=n_2\widehat{dx}^{n-2}d^2x$$

$$X_2=n_3\widehat{dx}^{n-3}d^3x+3n_4\widehat{dx}^{n-4}(d^2x)^2=\overset{1}{\underset{c}{d}}(\underset{1}{\delta}X_1)=(\underset{c}{d}\underset{1}{\delta})^2X_0,$$

$X_3 = n_4 \widehat{dx}^{n-4} d^4x + 10 n_5 \widehat{dx}^{n-5} d^2x d^3x + 15 n_6 \widehat{dx}^{n-6} (d^2x)^3 = \overset{1}{\underset{c}{d}}(\partial X_2)$,

$X_4 = n_5 \widehat{dx}^{n-5} d^5x + 10 n_6 \widehat{dx}^{n-6} \widehat{d^3x}^2 + 15 n_6 \widehat{dx}^{n-6} d^2x d^4x + 105 n_7 . \widehat{dx}^{n-7} \widehat{d^2x}^2 d^3x +$

$+ 105 . n_8 \widehat{dx}^{n-8} \widehat{d^2x}^4 = \overset{1}{\underset{c}{d}}(\underset{1}{\partial} X_3)$, etc., generatim $X_r = (\overset{1}{\underset{c}{d}} \underset{1}{\partial})^r X_0$, seu

$X_1 = n_2 \widehat{dx}^{n-2} d^2x$

$$X_2 = n_3 dx^{n-4} . \left(dx . d^3x + 3 . \frac{n-3}{4} . \widehat{d^2x}^2 \right).$$

$$X_3 = n_4 dx^{n-6} . \left(\widehat{dx}^2 d^4x + 10 . \frac{n-4}{5} . dx d^2x d^3x + 15 . \frac{n-4}{5} . \frac{n-5}{6} . \widehat{d^2x}^3 \right)$$

generatimque $X_{r+1} = \overset{1}{\underset{c}{d}}(\underset{1}{\partial} X_r)$. Hinc igitur habetur formula generalis ipsius $d^n f = X_0 \partial^n f + X_1 \partial^{n-1} f + X_2 \partial^{n-2} f + X_3 \partial^{n-3} f + \ldots + X_{n-2} \partial^2 f + X_{n-1} \partial f$, seu $d^n f = \widehat{dx}^n . \partial^n f + n_2 dx^{n-2} d^2x \partial^{n-1} f + (n_3 \widehat{dx}^{n-3} d^3x + 3 n_4 \widehat{dx}^{n-4} (d^2x)^2) \partial^{n-2} f +$ &c. $+ d^n x . \partial f$, quæ differentiale cujusvis ordinis breviter exprimit.

Ex. gr. Ponatur $n = 6$, eritque

$$d^6 f = \widehat{dx}^6 \partial^6 f + 15 \widehat{dx}^4 d^2x . \partial^5 f + (45 \widehat{dx}^2 \widehat{d^2x}^2 + 20 dx^3 d^3x) \partial^4 f +$$
$$+ (15 \widehat{d^2x}^3 + 60 dx d^2x d^3x + 15 \widehat{dx}^2 d^4x) \partial^3 f +$$
$$+ (10 \widehat{d^3x}^2 + 15 d^2x d^4x + 6 dx d^5x) \partial^2 f + d^6x . \partial f;$$

quod etiam differentiale ex regulis antea traditis, ipsum $d^5 f$ jam exhibitum singulari nostro modo differentiando, facile obtinebis.

8) Neque silentio prætereundus est **alius modus**, hos coefficientes vere differentiales X_0, X_1, X_2, ... X_{n-1}, X_n definiendi, idque tanto minus, cum ipse ab ultimis X_n, X_{n-1} aptissime initium faciat. Hic modus scilicet seorsim differentialium complexus seorsimque horum coefficientes efformat. Ipsi vero complexus in $d^n f$ præter derivata functionis f differen-

tialia habent ipsius x, quorum exponentes, interni cum externis, ex *combinatione ad summam n* oriuntur. Sic Ex. gr. pro $n=6$, complexus combinationum ad summam 6, sunt $^{1})$ 6; $^{2})$ 15, 24, 33; $^{3})$ 114, 123, 222; $^{4})$ 1113, 1122; $^{5})$ 11112; $^{6})$ 111111; indeque hi differentialium oriuntur acervi (cum $d^1x=dx$ sit): 1) d^6x; 2) dxd^5x, d^2xd^4x, $\widehat{d^3x}^2$; 3) $\widehat{dx}^2d^4x$, dxd^2xd^3x, $\widehat{d^2x}^3$; 4) $\widehat{dx}^3d^3x$, $\widehat{dx}^2\widehat{d^2x}^2$; 5) $(dx)^4d^2x$; 6) $(dx)^6$; quorum diversi ordines (seu *classes* ut vulgo audiunt) ad suam quique ipsius f functionem derivatam cum numero justo N juncti, totum d^6f constituent: nempe

$$d^6f=d^6x.\partial f+(N\widehat{1,5}.dxd^5x+N\widehat{2,4}.d^2x.d^4x+N\widehat{3^2}.\widehat{d^3x^2})\partial^2 f+$$
$$+(N\widehat{1^2,4}.\widehat{dx}^2d^4x+N\widehat{1,2,3}.dxd^2xd^3x+N2^3.\widehat{d^2x}^3)\partial^3 f+$$
$$+(N\widehat{1^3,3}.\widehat{dx}^3d^3x+N\widehat{1^2,2^2}.\widehat{dx}^2d^2x)\partial^4 f+$$
$$+N\widehat{1^4,2}\widehat{dx}^4d^2x\partial^5 f+\widehat{dx}^6\partial^6 f.$$

Ejusmodi vero complexus ex regulis notissimis doctrinæ combinationum efformantur. (Ex. gr. ${}^1C^n=n$; ${}^2C^n=1\,\overline{n-1}$, $2\,\overline{n-2}$, ... $r\,\overline{n-r}$ .... seu ${}^2C^n=1.{}^1C^{n-1}+2.{}^1C^{n-2}+3.{}^1C^{n-3}+\ldots$ (= classis secunda combinationum ad summam n); ${}^3C^n=1.{}^2C^{n-1}+2.{}^2C^{n-2}+3.{}^2C^{n-3}+\ldots$ seu ${}^3C^n=11\,\overline{n-2}$, $12\,\overline{n-3}$, $13\,\overline{n-4}$ ....; $22\,\overline{n-4}$, $23\,\overline{n-5}$, etc......).

Quod vero ad numeros attinet, hi tum diverso modo ex sese tum absolute formantur. Sit nempe

$N(1^\alpha,2^\beta,3^\gamma \ldots r^\varrho).\widehat{dx}^\alpha.\widehat{d^2x}^\beta.\widehat{d^3x}^\gamma \ldots \widehat{d^rx}^\varrho\partial^{n-r}f$ terminus aliquis in d^nf, similisque

$M(1^{\alpha'},2^{\beta'},3^{\gamma'},\ldots r^{\varrho'},)\widehat{dx}^{\alpha'}.\widehat{d^2x}^{\beta'}.\widehat{d^3x}^{\gamma'} \ldots \widehat{d^rx}^{\varrho'}.\partial^{n-r'}f$ in $d^{n-1}f$;

Eritque, cum $d^nf=n\overset{1}{\underset{c}{d}}(d^{n-1}f)$ sit,

$$n.\overset{1}{\underset{c}{d}}(M\overline{1^\alpha,2^\beta,3^\gamma,\ldots \overline{r-2}^{\nu},r^{\varrho'}}.\widehat{dx}^\alpha\widehat{d^2x}^\beta\widehat{d^3x}^\gamma \ldots \overline{d^{r-2}x}^{\nu}.\widehat{d^rx}^{\varrho'})\partial^{n-r}f=$$

$$=n.M\overline{1^\alpha,2^\beta,3^\nu\ldots\overline{r-2}^\nu,r\varrho'}.\widehat{dx^\alpha}.\widehat{d^2x^\beta}\widehat{d^3x^\gamma}\ldots\widehat{d^{r-2}x^\nu}.\varrho'\widehat{d^rx^{\varrho'-1}}\,\frac{d^{r+1}x}{r+1}.\delta^{n-r}f$$ æquale termino correspondenti in d^nf, ideoque

$$N\overline{1^\alpha,2^\beta,3^\gamma\ldots\overline{r-2}^\nu,r\varrho'^{-1},r+1}=\frac{n\varrho'}{r+1}.M\overline{1^\alpha,2^\beta,3^\gamma\ldots\overline{r-2}^\nu,r\varrho'}$$

seu $N\overline{1^\alpha,2^\beta,3^\gamma\ldots\overline{r-2}^\nu,r\varrho,r+1}=n.\frac{\varrho+1}{r+1}.M\overline{1^\alpha,2^\beta,3^\gamma,\ldots\overline{r-2}^\nu,r\varrho^{+1}}$

ideoque numerus N(in d^nf seu ad exponentem n pertinens) hujus formulæ ope ex coefficiente M, ad exponentem $(m=)$ $n-1$ pertinente, computatur. Sit vero $\alpha+2\beta+3\gamma+\ldots+\overline{r-2}\,\nu+r\varrho+r+1=n$, qui numerus combinationum constituebat summam.

Demum observandum est, coefficientem $N\overline{1^\alpha2^\beta3^\gamma\ldots\ldots}$ absolute hunc in modum exprimi:

$$N\,\overline{1^\alpha2^\beta3^\gamma}\ldots=\frac{\Gamma\overline{\alpha+2\beta+3\gamma+\ldots}}{\Gamma\alpha.\Gamma\beta.\Gamma\gamma\ldots\Gamma1^\alpha.\Gamma2^\beta.\Gamma3^\gamma\ldots}\,,\quad\text{existente}\quad\Gamma x=1.2.3\ldots.x,$$

$(\Gamma1=1=\Gamma o)$, atque $\alpha+2\beta+3\gamma+\ldots=n$. Ex. gr. $N(3^2)=\frac{\Gamma6}{\Gamma2.\Gamma3^2}=\frac{4.5}{2}=10$.

Cum igitur ${}^1C^n=n$, ${}^2C^n=1\,\overline{n-1},2\,\overline{n-2},3\,\overline{n-3},\ldots{}^3C^n=1\,{}^2C^{n-1}+2\,{}^2C^{n-2}+\ldots$ sit, atque symbolice $X_n=(\delta)\,{}^1C^n(x)$, $X_{n-r}=(\delta)\,{}^2C^n(x)$, etc.; erit

$$d^nf=d^nx.\delta f+(N\overline{1^1\overline{n-1}^1}dxd^{n-1}x+N\overline{2^1\overline{n-2}^1}d^2xd^{n-2}x+\ldots)\delta^2f+$$

$$+[N\overline{1^2\overline{n-2}}.dx^2.d^{n-2}x+N\overline{1^12^1\overline{n-3}^1}.dxd^2xd^{n-3}x+\ldots$$

$$N\overline{1^1\,3^1\,\overline{n-4}}\,dxd^3xd^{n-4}x+N\overline{2^2\,\overline{n-4}}.\widehat{d^2x}^2d^{n-4}x+$$

$$+N\overline{2^13^1\overline{n-5}^1}\,d^2xd^3xd^{n-5}x+\ldots\ldots)\ \delta^3f+\ldots\quad\text{seu}$$

$$d^nf=d^nx.\delta f+\left(\frac{\Gamma n}{\Gamma1.\Gamma\overline{n-1}}.dxd^{n-1}x+\frac{\Gamma n}{\Gamma\overline{n-2}.\Gamma2}.d^2xd^{n-2}x+\ldots\right)\delta^2f+\ldots$$

$$+\left(\frac{\Gamma n}{\Gamma 2.\Gamma\overbrace{n-2}}dx^2d^{n-2}x+\frac{\Gamma n}{\Gamma 2.\Gamma\overbrace{n-3}}dxd^2xd^{n-3}x+\ldots\right)\partial^3 f+\ldots,\quad \text{seu}$$

$$d^n f=d^n x.\partial f+(ndxd^{n-1}x+n_2d^2xd^{n-2}x+n_3d^3xd^{n-3}x+\ldots.)\ \partial^2 f+$$
$$+(n_2dx^2d^{n-2}x+3n_3dxd^2xd^{n-3}x+\ldots.)\ \partial^3 f+\ldots.$$

His jam regulis effectum est, ut $d^n(fx)$ pro quocunque valore integro ipsius (n) vel immediate in terminis explicitis absque ulla reductione habeatur, vel successive ex differentialium anteriorum valoribus explicitis et immediate reductis evolvatur, ut hac parte parum ulterius desiderari vel superaddi posse videatur. Nostrarum vero regularum, quas juvenes conscripsimus, *demonstratio* ex theoria derivationis ampliori et disciplina combinationum peti potest, heic vero utut satis prolixa, prætermissa est. Simplicius forsan ita hæc res efficiatur, ut primum observetur differentiationem completam, coefficientibus constantibus sepositis, alios haud præbere terminos variabiles, ac quæ differentiatio secundum duo altissima in unoquoque termino præsentia sibique consequentia differentialia suppeditet; deinde eligatur functio specialis (ex. gr. potentia), cujus evolutione coefficientes determinari possint; Id quod aliis juvenibus disquirendum relinquimus. Nec spernendi erunt admininiculi *Hindenburgianæ* de polynomio elevando regulæ; quas igitur interim consulas.

Patet enim has res arcte cohærere. Vidimus enim antea, hanc evolutionem

$\varphi(z+rdz+r^2\underset{c}{d}^2z+\ldots)$, quam per

$\varphi z+rd\varphi z+r^2\underset{c}{d}^2\varphi z+r^3\underset{c}{d}^3\varphi z+\ldots$ indicavimus, ad hanc revocari

$\varphi(z+r\zeta)=\varphi z+r\zeta\varphi_1 z+r^2\zeta^2\varphi_2 z+r^3\zeta^3\varphi_3+\ldots$, ubi

$\zeta=dz+r\underset{\circ}{d}^2z+r^2\underset{\circ}{d}^3z+\ldots$, ut igitur totum negotium ad potentiam

$\zeta^n = (z_1 + rz_2 + r^2z_3 + \ldots)^n$ rite evolvendam redeat, de qua & Hindenburg egit & nobis seorsim agendi est animus.

Coronidis

Loco *novum & analysin & ipsam functionum evolutionem considerandi modum* breviter indicare lubet.

Jam enim Præs. partim in tractatu Gothoburgi eruditis biennio ante exhibito partim in lectionibus tam publicis quam privatis monstravit, ipsam operationem Matheseos primariam, a qua ceteræ originem ducunt, *additionem* dico, multo generalius concipi posse, ut ea sit, quæ hac gaudeat proprietate, $(x|y)|z = x|y|z = (z|y)|x$, exsistentibus x, y & z quibusvis, prætereaque $x|\mathfrak{Z} = x$, exsistente $\mathfrak{Z}$ determinato quodam, huic operationi *indifferente*. Talia vero *aggregata* $x|y$ innumera dantur, ut $x+y+axy$, $\frac{x+y+axy}{1-cxy}$ &c. Viceversa vero si $x|y = z$, erit x *differentia formalis* (si $z\,\underline{|y}$ vel $z|\bar{y}$ vel $z\,\Gamma y$) ipsorum z & y.

Quibus positis, *producta generaliora* $x|x = 2\odot x$, $x|x|x = 3\odot x$, $x|x|x|\ldots|x = n\odot x$, seu $(n+1)\odot x = (n\odot x)|x$ per aggregata illa explicantur, proprietatesque ipsis producti communis simillimas his vindicantur, idque eodem fere modo, ac quo *doctrina proportionum* stabilitur:

$$\text{Ut } n\odot(a|b) = \overbrace{n\odot a}\,|\,\overbrace{n\odot b}$$

$$n\odot(r\odot a) = r\odot(n\odot a) = (n.r)\odot a$$

$$n\odot(a\,\Gamma b) = \overbrace{n\odot a}\,\Gamma\,\overbrace{n\odot b}$$

$$\overbrace{n \odot a} \,|\, \overbrace{r \odot a} = (n+r) \odot a$$

$$\overbrace{n \odot a} \,\Gamma\, \overbrace{r \odot a} = \overbrace{n - r} \odot a.$$

Et si $x \odot y = z$ fuerit, erit invertendo $x = z\, \overline{\odot\, y}$ vel $= z \odot^{-} y$ *Hyperlogarithmus* seu *exponens generalis* ipsorum z & y, atque $y = x \odot z$ *Antilogarithmus functionalis* ipsorum x & z. De quibus formulæ istis correspondentes valent. Jam vero, quia aggregatum hoc generale $x \,|\, y$ eadem gaudet proprietate $x|y|z = (x|y)|z = x|(y|z)$, qua demonstratio Theorematis Taylorei nititur (ut p. 4 sq. vidisti); eodem fere modo simile, sed multo latius patens *Theorema* demonstratur, quod ita sonat:

$F(x|y) = Fx|(\Lambda y \odot \mathfrak{D} Fx)|\widehat{\Lambda y}^2 \odot \underset{c}{\mathfrak{D}}^2 Fx|\widehat{\Lambda y}^3 \odot \underset{c}{\mathfrak{D}}^3 Fx|..$, exsistente Λy certa ad ipsum aggregatum ($|$) ita relata functione, ut sit $\Lambda(x|y) = \Lambda x + \Lambda y$, (ideoque ipsius hyperlogarithmus), atque $\mathfrak{D} Fx$ functione hoc ipso novo maximoque generali evolvendi modo ex functione F (cum Λ) *derivata*; ceteris vero derivatis $\underset{c}{\mathfrak{D}}^2 Fx$, $\underset{c}{\mathfrak{D}}^3 Fx$ &c. ex prima ita generatis, ut sit

$$\underset{c}{\mathfrak{D}}^2 Fx = \tfrac{1}{2} \odot \mathfrak{D}(\mathfrak{D} Fx) = \tfrac{1}{2} \odot \mathfrak{D}^2 Fx,\ \underset{c}{\mathfrak{D}}^3 Fx = \tfrac{1}{2} \odot \tfrac{1}{3} \odot \mathfrak{D}\mathfrak{D}^2 Fx = \tfrac{1}{2.3} \odot \mathfrak{D}^3 Fx,$$

universimque $\underset{c}{\mathfrak{D}}^n Fx = \dfrac{1}{\Gamma n} \odot \mathfrak{D}^n Fx$, exsistente $\Gamma n = 1.2.3 \ldots n = n.\Gamma\overline{n - 1}.$

Speciatim igitur si $x|y = x + y$ posueris, ipsum Theorema Tayloreum inde facile colliges. Sin vero $x|y = x + y + axy$ & $\Lambda x = L(1 + ax)$ posueris, aliud Theorema tibi constabit, similiterque innumera isti æmula colliges. Tota vero hæc res melius in *Theoria functionum iteratarum* explicatur.